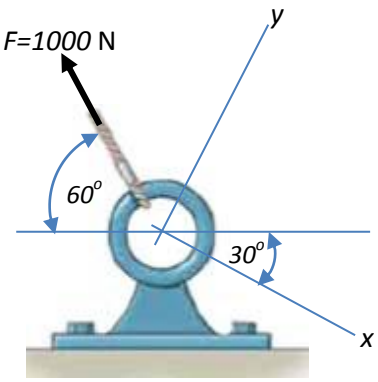
Mansoura University | **Engineering Mechanics I**
Faculty of Engineering | **First term (2011-2012)**
CIE Program. | **Final Exam** | **Time**: Two hours

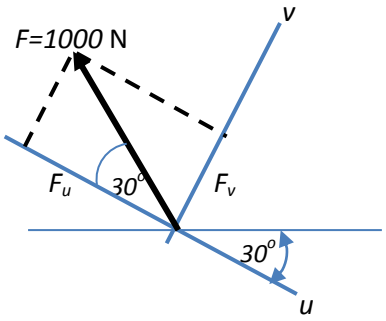
Instructor: ***Prof. Dr. Bishri Abdel-Mo'emen***

Answer the Following Questions (Total mark 50 points)

Question 1: (5 Points)

Determine the x and y components of the 1000 N force shown in Figure.



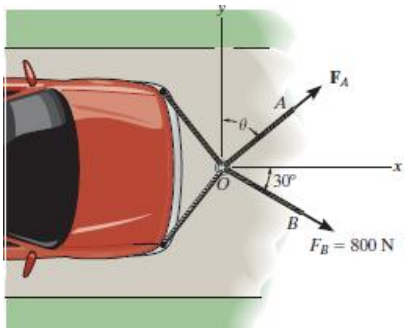


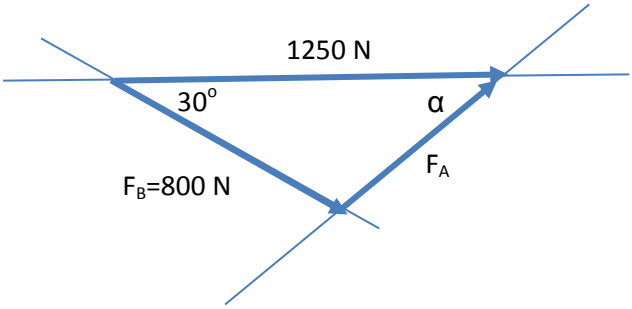
$F_u = -1000 \cos 30^o$
$= -866$ N

$F_v = 1000 \sin 30^o$
$= 500$ N

Question 2: (5 Points)

Determine the magnitude of $\mathbf{F}_A$ and its direction θ so that the resultant force is directed along the positive x axis and has a magnitude of 1250 N.





$$F_A = \sqrt{(1250)^2 + (800)^2 - 2(1250)(800)cos\ 30}$$

F_A = 686 N

$$\frac{F_B}{sin\ \alpha} = \frac{F_A}{sin\ 30}$$

$$\alpha = sin^{-1}\left(\frac{F_B\ sin\ 30}{F_A}\right) = 35.7$$

$$\theta = 90 - \alpha = 54.3^o$$

Another Solution:

$R_x = \sum F_x$; 1250 = F_A sin θ +800 cos 30°

F_A sin θ = 557.2 (a)

$R_y = \sum F_y$; 0 = F_A cos θ -800 sin 30°

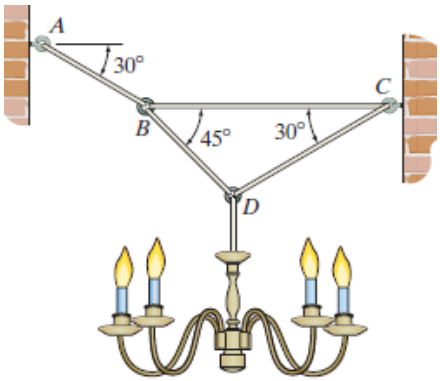
F_A cos θ = 400 (b)

Solving (a) and (b), gives :

F_A= 686 N and θ = 54.3°

Question 3: (8 Points)

Determine the tension developed in each rope used to support the 40-kg chandelier.



Equilibrium of Point D*:*

$$\sum F_x = 0; \quad F_{CD}\cos 30 - F_{DB}\cos 45 = 0$$

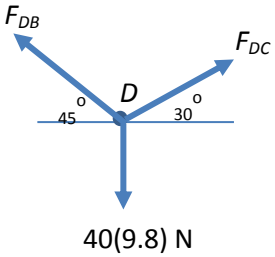
$$F_{CD} = \frac{F_{DB}\cos 45}{\cos 30} = 0.816\,F_{DB} \qquad \text{------ (1)}$$

$$\sum F_y = 0; \quad F_{CD}\sin 30 + F_{DB}\sin 45 - 40(9.8) = 0 \qquad \text{------ (2)}$$

Solving (1) and (2) : $0.408\,F_{DB} + 0.707\,F_{DB} = 392$

Thus, $F_{DB} = 351.6\ N$ and $F_{CD} = 286.9\ N$

Equilibrium of Point B*:*



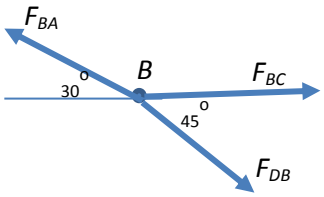
$$\sum F_x = 0; \quad F_{BC} - F_{BA}\cos 30 + F_{BD}\,cos45 = 0$$

$$F_{BC} = 0.866\,F_{BA} - 0.707\,F_{BD} \qquad \text{------ (3)}$$

$$\sum F_y = 0; \quad F_{BA}\sin 30 - F_{DB}\sin 45 = 0 \qquad \text{------ (4)}$$

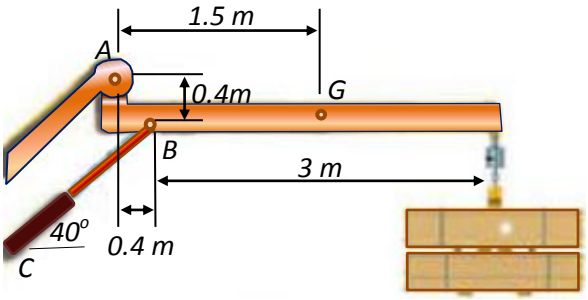
Solving (3) and (4): $F_{BA} = 1.414\,F_{DB} = 497.2\ N$

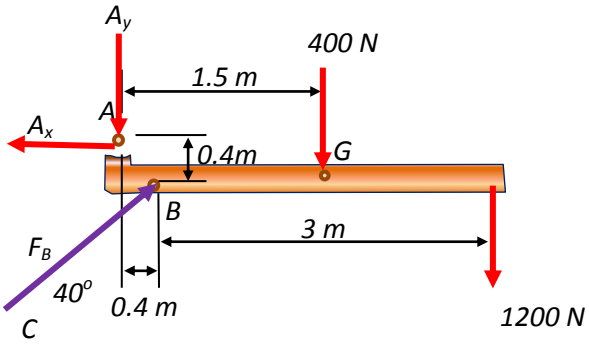
$F_{CB} = 182\ N$



Question 4: (8 Points)

The articulated crane boom has a weight of 400 N and a center of gravity at *G*. If it supports load of 1200 N, determine the force acting at the pin *A* and the force in the hydraulic cylinder *BC* when the boom is in the position shown.





Question 4: Solution

$\sum_{\square} M_A = 0;$

$$F_B \cos 40 * (0.4) + F_B \sin 40 * (0.4) - 400 * (1.5) - 1200 * (3.4) = 0$$

$$F_B = 8305\ N.m \qquad Ans$$

$\sum_{\square} F_x = 0;$ $\quad -A_x + F_B \cos 40 = 0$

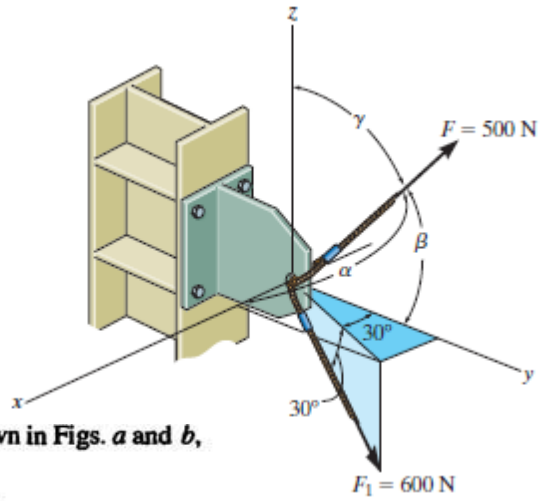
$$\rightarrow \qquad A_x = 6362\ N \qquad Ans$$

$\sum_{\square} F_y = 0;$ $\quad -A_y + F_B \sin 40 - 400 - 1200 = 0$

$$\rightarrow \qquad A_y = \ 3738.4\ N \qquad Ans$$

Question 5: (5 Points)

Two forces **F** and $\mathbf{F}_1$ act on a bracket as shown in Figure. If the resultant force of **F** and $\mathbf{F}_1$ is directed along the positive *y* axis, determine the magnitude of the resultant force and the coordinate direction angles of **F** so that $\beta < 90^o$.



Force Vectors: By resolving $\mathbf{F}_1$ and **F** into their *x*, *y*, and *z* components, as shown in Figs. *a* and *b*, respectively, $\mathbf{F}_1$ and **F** can be expressed in Cartesian vector form as

$$\mathbf{F}_1 = 600\cos 30°\sin 30°(+\mathbf{i}) + 600\cos 30°\cos 30°(+\mathbf{j}) + 600\sin 30°(-\mathbf{k})$$
$$= \{259.81\mathbf{i} + 450\mathbf{j} - 300\mathbf{k}\}\ \text{N}$$
$$\mathbf{F} = 500\cos\alpha\mathbf{i} + 500\cos\beta\mathbf{j} + 500\cos\gamma\mathbf{k}$$

Since the resultant force $\mathbf{F}_R$ is directed towards the positive *y* axis, then

$$\mathbf{F}_R = F_R\mathbf{j}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}$$
$$F_R\mathbf{j} = (259.81\mathbf{i} + 450\mathbf{j} - 300\mathbf{k}) + (500\cos\theta_x\mathbf{i} + 500\cos\theta_y\mathbf{j} + 500\cos\theta_z\mathbf{k})$$
$$F_R\mathbf{j} = (259.81 + 500\cos\alpha)\mathbf{i} + (450 + 500\cos\beta)\mathbf{j} + (500\cos\gamma - 300)\mathbf{k}$$

Equating the **i**, **j**, and **k** components,

$$0 = 259.81 + 500\cos\alpha$$
$$\alpha = 121.31° = 121° \qquad \textbf{Ans.}$$
$$F_R = 450 + 500\cos\beta \qquad (1)$$

$$0 = 500\cos\gamma - 300$$
$$\gamma = 53.13° = 53.1° \qquad \textbf{Ans.}$$

However, since $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, $\alpha = 121.31°$, and $\gamma = 53.13°$,

$$\cos\beta = \pm\sqrt{1 - \cos^2 121.31° - \cos^2 53.13°} = \pm 0.6083$$

If we substitute $\cos\beta = 0.6083$ into Eq. (1),

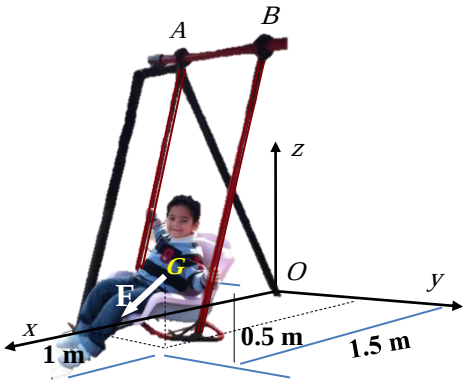
$$F_R = 450 + 500(0.6083) = 754\ \text{N} \qquad \textbf{Ans.}$$

and

$$\beta = \cos^{-1}(0.6083) = 52.5° \qquad \textbf{Ans.}$$

Question 6: (5 Points)

A boy exerts a force F= (20 i – 200 k) N on a swing as shown. The force acts at the boy's center of mass *G*. Determine the moment created by *F* about the axis *AB*. The coordinates of bearings at *A* is (1,0.5, 2.5) and at bearing *B* is (1, 1.5, 2.5).



Given: A: (1,0.5, 2.5) ; B: (1, 1.5, 2.5); F= (20 **i** – 200 **k**) N

G: (1.5,1, 0.5) ,

r_{AG}=0.5i+0.5j-2k

$$\mathbf{M}_A = \mathbf{r}_{AG} \times \mathbf{F}$$

$$\mathbf{M}_A = \begin{vmatrix} i & j & k \\ 0.5 & 0.5 & -2 \\ 20 & 0 & -200 \end{vmatrix}$$

= -100i – (-100+40)j+(-10 k)

= (-100i + 60j -10 k) N.m

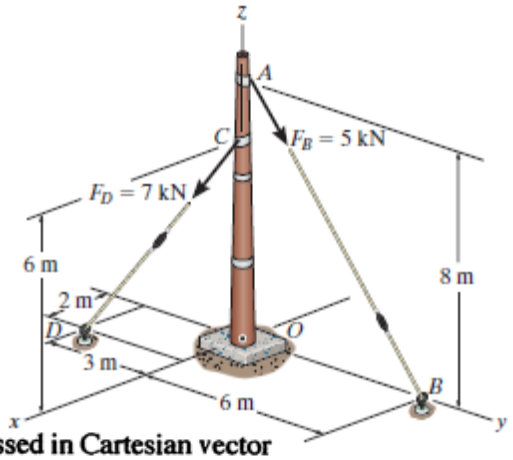
The axis *AB* is parallel to y-axis. So, the moment about the axis is: 60 N.m.

OR

$\mathbf{M}_{AB} = \mathbf{M}_A.\mathbf{U}_{AB}$ = (-100**i** + 60**j** -10 **k**).(**j**) = 60 N.m

Question 7: (8 Points)

Two forces act on the post as shown in Figure. Replace the two forces by an equivalent force-couple system at point *O*.



Equivalent Resultant Force: The forces $\mathbf{F}_B$ and $\mathbf{F}_D$, Fig. *a*, expressed in Cartesian vector form can be written as

$$\mathbf{F}_B = F_B\mathbf{u}_{AB} = 5\left[\frac{(0-0)\mathbf{i}+(6-0)\mathbf{j}+(0-8)\mathbf{k}}{(0-0)^2+(6-0)^2+(0-8)^2}\right] = [3\mathbf{j}-4\mathbf{k}]\text{kN}$$

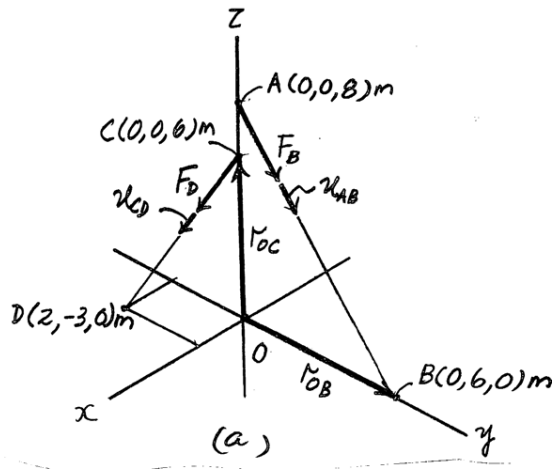
$$\mathbf{F}_D = F_D\mathbf{u}_{CD} = 7\left[\frac{(2-0)\mathbf{i}+(-3-0)\mathbf{j}+(0-6)\mathbf{k}}{(2-0)^2+(-3-0)^2+(0-6)^2}\right] = [2\mathbf{i}-3\mathbf{j}-6\mathbf{k}]\text{kN}$$

The resultant force $\mathbf{F}_R$ is given by

$$\mathbf{F}_R = \Sigma\mathbf{F};\quad \mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_D$$
$$= (3\mathbf{j}-4\mathbf{k})+(2\mathbf{i}-3\mathbf{j}-6\mathbf{k})$$
$$= [2\mathbf{i}-10\mathbf{k}]\text{kN} \qquad \textbf{Ans.}$$

Equivalent Resultant Force: The position vectors $\mathbf{r}_{OB}$ and $\mathbf{r}_{OC}$ are

$$\mathbf{r}_{OB} = \{6\mathbf{j}\}\text{ m} \qquad \mathbf{r}_{OC} = [6\mathbf{k}]\text{ m}$$



Thus, the resultant couple moment about point *O* is given by

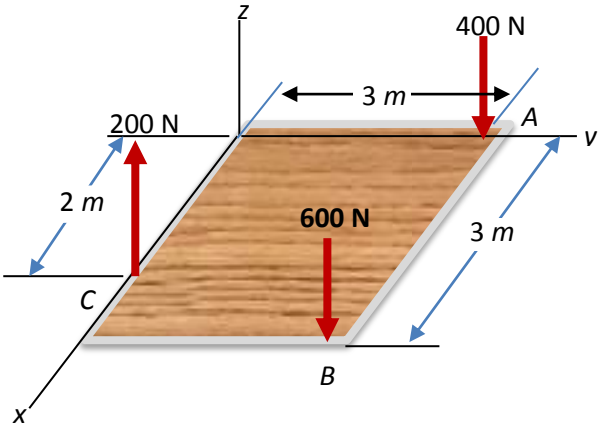
$$(\mathbf{M}_R)_O = \Sigma\mathbf{M}_O;\qquad (\mathbf{M}_R)_O = \mathbf{r}_{OB}\times\mathbf{F}_B + \mathbf{r}_{OC}\times\mathbf{F}_D$$
$$= \begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\0&6&0\\0&3&-4\end{vmatrix} + \begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\0&0&6\\2&-3&-6\end{vmatrix}$$
$$= [-6\mathbf{i}+12\mathbf{j}]\text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

$$M_{R_A} = \Sigma M_A;\quad 10750d = -3500(3)-5500(17)-1750(25)$$

$$d = 13.7\text{ ft} \qquad \textbf{Ans.}$$

Question 8: (8 Points)

Replace the loading shown by an equivalent single resultant force and specify the x and y coordinates of its line of action.



A: (0, 3, 0) ; *B:* (3, 3, 0); *C*: (2, 0, 0)

The Equivalent system at the Origin:

1. Summing the moment about the origin:

   M= 3**j** ^ (-400 **k) + (**3**i**+3**j** ^ (-600 **k) +** 2**i** ^ (200 **k)**

   = -1200 **i** + (1800 **j** – 1800 **i)** – 400 **j**

   = -3000 **i** + 1400 **j**

2. The resultant : **R =** -400 **k** -600 **k** + 200 **k** = -800 **k**

   The Single Resultant is : **R** = -800 **k** and acts at point (*x, y, 0*), where:
   (*x* **i** + *y* **j**)^ (**R) = M**
   (*x* **i** + *y* **j**)^ (-800 **k) =** -3000 **i** + 1400 **j**

   800 *x* **j** – 800 *y* **i=** -3000 **i** + 1400 **j**

   x = (1400 /800) = 1.75 m

and *y = (3000 /800) = 3.75 m*

Good Luck and Happy New Year

Dr. Bishri Abdel-Mo'emen