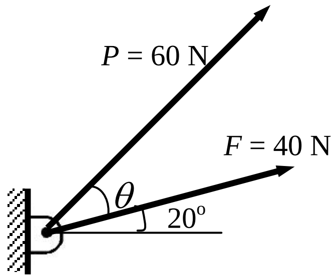
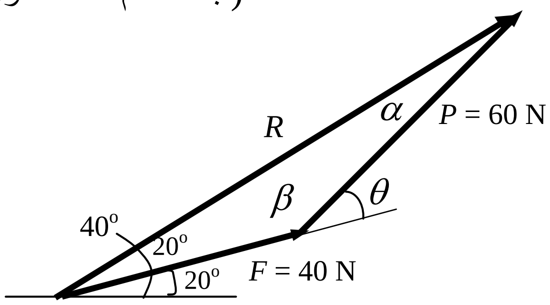
Final ExamSolution - *First Semester* *2002-2003*

Question (1): *(10 Points)*

If the resultant of the two forces shown in Figure makes 40° with the positive *x*-direction measured counter-clockwise, determine the magnitude of the resultant and the angle θ between the two forces **P** and **F**.



Solution I (باستخدام مثلث القوى)



$$\frac{P}{\sin 20} = \frac{F}{\sin \alpha} \rightarrow \sin \alpha = \frac{F}{P} \sin 20$$

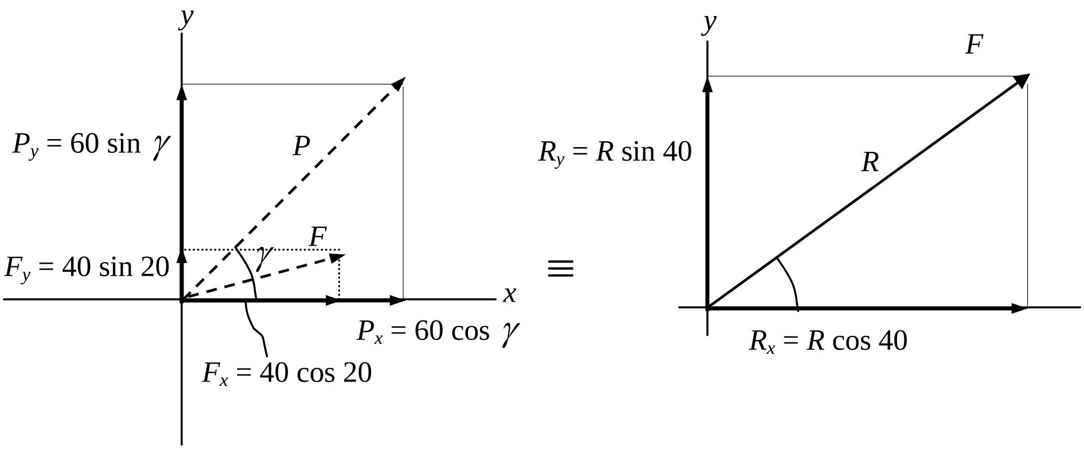
$$\alpha = \sin^{-1}\left(\frac{40}{60} \sin 20\right) = 13.18^{o}$$

$$\therefore \quad \theta = \alpha + 20^{o} = \boxed{\mathbf{33.18^{o}}} \quad \Leftarrow \textbf{ Answer}$$

$$\frac{R}{\sin \beta} = \frac{P}{\sin 20} \rightarrow R = \frac{P}{\sin 20} \sin \beta$$

$$= \frac{60}{\sin 20} \sin(180 - 33.18) = \boxed{\mathbf{96\ N}} \quad \Leftarrow \textbf{ Answer}$$

Solution II (بالتحليل الى مركبات)



$$R_x = F_x + P_x \quad \rightarrow \quad R \cos 40 = 40 \cos 20 + 60 \cos \gamma \qquad \ldots\ldots \text{(a)}$$

$$R_y = F_y + P_y \quad \rightarrow \quad R \sin 40 = 40 \sin 20 + 60 \sin \gamma \qquad \ldots\ldots \text{(b)}$$

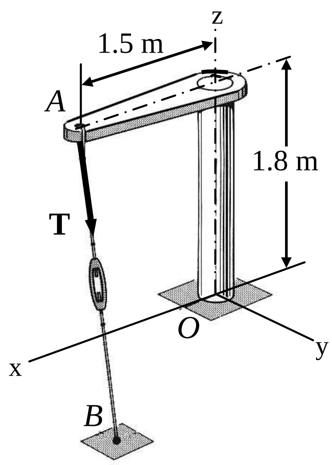
Squaring (a) and (b) and adding the results

$$R^2 = (40)^2 + (60)^2 + 2(40)(60)\,[\cos 20 \cos \gamma + \sin 20 \sin \gamma]$$

$$= (40)^2 + (60)^2 + 2(40)(60) \cos (20 + \gamma)$$

Question (2): *(1 · Points)*

The tension in cable *AB* is **T**=300**i**+800**j**–1200**k**. Resolve this tension force into three orthogonal components **F**, **P**, and **W** defined as follows: Force **F** is parallel to the line joining *AO*, force **P** is to be parallel to the *xy*-plane, and force **W** is perpendicular to both **F** and **P**.



The given tension is: **T**=300**i**+800**j**–1200**k**

i. The component **F**: (*along AO*)

$\mathbf{r}_{AO}$ = -1.5**i** - 1.8 **k**,

$r_{AO} = \sqrt{1.5^2 + 1.8^2}$ =2.34 m

$\therefore\ \mathbf{u}_{AO} = \dfrac{\mathbf{r}_{AO}}{r_{AO}} =$ -0.64**i** – 0.768 **k**

$\therefore\ \mathbf{F} = (\mathbf{T}.\mathbf{u}_{AO})\,\mathbf{u}_{AO} = 729.6\,\mathbf{u}_{AO}$

= -466.94**i** – 560.33.8 **k** *Ans*

ii. The component **P**: *(Parallel to the xy-plane)*

Assume a position vector parallel to the *xy*-plane:

$\mathbf{r}_p$ = x**i** + y **j**,

If **r** is perpendicular to $\mathbf{r}_{AO}$, then $\mathbf{r}_p.\mathbf{r}_{AO} = 0$,

$\therefore$ -1.5x = 0, → x=0 → $\therefore\ \mathbf{r}_p = y\mathbf{j}$ → $\therefore\ \mathbf{u}_p = \mathbf{j}$

$\therefore\ \mathbf{P} = (\mathbf{T}.\mathbf{u}_p)\,\mathbf{u}_p = 800\,\mathbf{u}_p =$ 800 **j** *Ans*

iii. The component **W**: *(Normal to both* **F** *and* **P***)*

This component is parallel to the vector

$\mathbf{r}_n = \mathbf{r}_{AO} \times \mathbf{u}_p = (-1.5\mathbf{i} - 1.8\,\mathbf{k}) \times \mathbf{j} = 1.8\,\mathbf{i} - 1.5\mathbf{k}$

$\therefore\ \mathbf{u}_n = \dfrac{\mathbf{r}_n}{r_n} = 0.768\,\mathbf{i} - 0.64\mathbf{k}$

$\therefore\ \mathbf{W} = (\mathbf{T}.\mathbf{u}_n)\mathbf{u}_n = 998.4\mathbf{u}_n = \boxed{766.77\mathbf{i} - 639.98\,\mathbf{k} \qquad \textit{Ans}}$

حل بمجرد النظر: يمكن بمجرد النظر ملاحظة الآتي:

١. جزء المستقيم *AO* يقع في مستوي *xz* وبالتالي أحد المركبات المطلوبة في اتجاه محور *y*

٢. حيث أن المركبة الثالثة عمودية أيضا على محور *y* فهي بالضرورة تقع في المستوي *xz* وفي نفس الوقت عمودية علي المستقيم *AO*.

٣. من المعلوم أنه إذا كانت مركبات المتجة هي $\mathbf{r} = a\mathbf{i} + b\mathbf{j}$ فإن مركبات المتجة العمودي عليه تكون $\mathbf{r}_n = -b\mathbf{i} + a\mathbf{j}$ أو $\mathbf{r}_n = b\mathbf{i} - a\mathbf{j}$

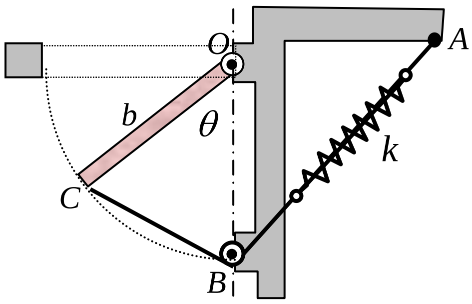
مما سبق يتضح مباشرة أن:

$\mathbf{r}_{AO} = -1.5\mathbf{i} - 1.8\,\mathbf{k},\ \ \mathbf{r}_n = 1.8\,\mathbf{i} - 1.5\mathbf{k}$, and $\mathbf{r}_p = \mathbf{j}$

وبذلك نحصل على المركبات الثلاثة حيث:

$\mathbf{F} = (\mathbf{T}.\mathbf{u}_{AO})\,\mathbf{u}_{AO}$, $\mathbf{P} = (\mathbf{T}.\mathbf{u}_p)\,\mathbf{u}_p$, and $\mathbf{W} = (\mathbf{T}.\mathbf{u}_n)\mathbf{u}_n$

Question (3): *(1٠ Points)*



The Figure shows a top view of a ventilator door of width *b* m hinged along its upper edge at *O* (مثبت عند *b* باب عرضه). The door closes automatically by the effect of a spring-loaded cable *ABC* that passes over a small frictionless pulley at *B*. The spring stiffness is *k* N/m and is unstretched when θ=0.

(السلك *ABC* يمر على بكره ملساء وبه جزء من سوستة تكون غير مشدودة عندما يكون الباب مغلق في الوضع *AB*)

a. Determine the restoring moment about the vertical axis (normal to paper) through *O* in terms of *b*, *k* and θ.
(إحسب عزم الإغلاق حول *O* بدلالة *b, k* و θ)

b. Find the magnitude of the moment if *b*=1.2m *k*=160 N/m and θ=60°.

a) The extension in the spring is:

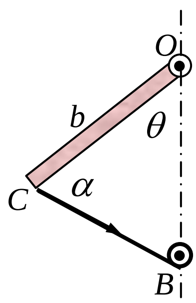
$$\Delta x = CB = b\sqrt{2(1-\cos\theta)} \rightarrow \text{(by the \textit{cosine law})}$$

The force in the spring is $F_s = k\Delta x = k\, b\sqrt{2(1-\cos\theta)}$

The moment about *O*:

$M_O = bF_s \sin\alpha$,

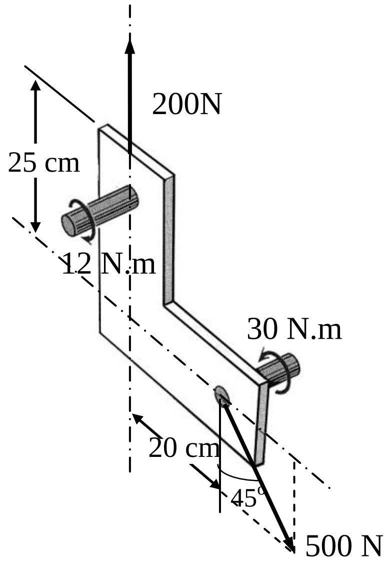
Using the *sine law:* $\sin\alpha = \dfrac{b}{CB}\sin\theta = \dfrac{\sin\theta}{\sqrt{2(1-\cos\theta)}}$



$\therefore \quad M_O = k\, b^2\sqrt{2(1-\cos\theta)}\dfrac{\sin\theta}{\sqrt{2(1-\cos\theta)}} = \boxed{k\, b^2 \sin\theta \quad Ans}$

b) When *b*=1.2 m, *k*=160 N/m, θ=60°, then

$M_O = 160\,(1.2)^2 \sin 60 = \boxed{199.53 \text{ N.m} \qquad Ans}$



Question (4): *(1 · Points)*

Replace the two forces and two couples acting on the rigid unit shown in Figure by an equivalent single resultant force and specify its location.

R_x = 500 cos 45 = 353.55 N

R_y = 200 –500sin 45 = -153.55 N

The magnitude of the resultant: $\boxed{R = 385.5\ N}$

The direction of the resultant: θ = $\tan^{-1}$(-153.55/353.55) = $\boxed{-23.5^{o}}$

Location of the resultant: Assume that **R** acts at the point (x, 0) on the x-axis, then Moment of **R** about O = Sum of moments about O.

$\therefore$ $-R_y$ (x) = -12 + 30 – (353.33)(0.2) = -52.666 N.m

$\therefore$ x = -52.666/(-153.55) = 0.343 m = 34.3 cm

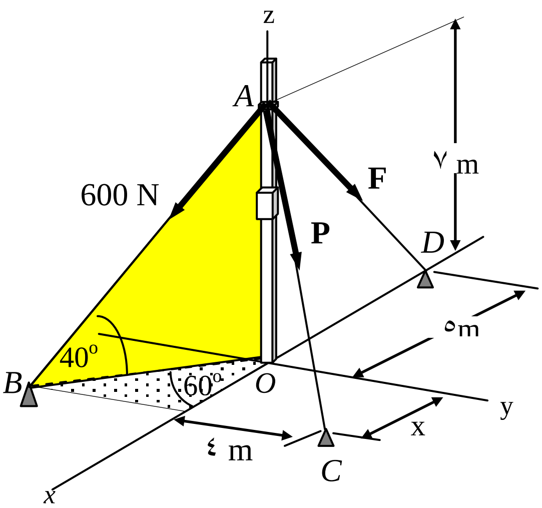
Thus, the resultant passes through $\boxed{(34.3, 0)}$

Question (5): *(1٥ Points)*

In preparation for the competitions of the All Universities Youth Week to be held at **Assut University** in February 2003, our university scouts team constructed a model of a boat sail using raw materials.

في إطار الإستعداد لفعاليات أسبوع شباب الجامعات الذي سوف يعقد بمشيئة الله في **جامعة اسيوط** خلال شهر فبراير ٢٠٠٣، فقد أعد فريق جوالة الجامعة نموذج لشراع قارب.

The sail mast (ساري الشراع) is supported by three cables as shown in Figure. The magnitude of the tension force in cable *AB* is 600N, in cable *AC* is P, and in cable *AD* is F. The length of cable *AC* is 8.6 m. It is required to locate point *C* so that the resultant of the forces in the three cables acts vertically downwards at point *A*. Determine the following:



a. The *x*-coordinate of point *C*.
b. The magnitudes P and F of the tensions in cables *AC* and *AD*.
c. The magnitude of the resultant forces.
d. The angle between cables *AB* and *AC*.
e. The moment of the tension in cable *AB* about point *C*.

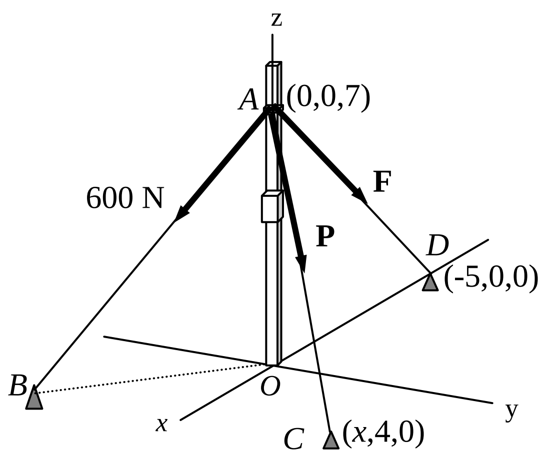
Solution

a) *The x-coordinate of point C*

r_{AC}=8.6 m (given), $\mathbf{r}_{AC} = (x\mathbf{i}+4\mathbf{j}-7\mathbf{k})$

$\therefore\ (8.6)^2 = x^2+(4)^2+(-7)^2$

$\therefore\ x = 2.99 \approx 3$ m   (***Ans***)



b) *The magnitudes P and F* :

$\mathbf{u}_{AB} = cos40\ cos60\ \mathbf{i} - cos40\ sin\ 60\ \mathbf{j} - sin\ 40\ \mathbf{k}$

$= 0.383\mathbf{i} - 0.663\mathbf{j} - 0.643\mathbf{k}$

$\therefore\ \mathbf{T}_{AB} = 600\ \mathbf{u}_{AB} = 600(0.383\mathbf{i} - 0.663\mathbf{j} - 0.643\mathbf{k})$

$= 229.8\mathbf{i} - 398\mathbf{j} - 385.67\mathbf{k},$

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{1}{8.6}(3\mathbf{i}+4\mathbf{j}-7\mathbf{k}) = 0.348\mathbf{i} + 0.465\mathbf{j} - 0.814\mathbf{k}$$

$\therefore\ \mathbf{P} = P\ \mathbf{u}_{AC} = P(0.348\mathbf{i} + 0.465\mathbf{j} - 0.814\mathbf{k}),$

$$\mathbf{u}_{AD} = \frac{\mathbf{r}_{AD}}{r_{AD}} = \frac{1}{8.6}(-5\mathbf{i} - 7\mathbf{k}) = -0.58\mathbf{i} - 0.814\mathbf{k}$$

$\therefore\ \mathbf{F} = F\ \mathbf{u}_{AD} = F(-0.58\mathbf{i} - 0.814\mathbf{k}),$

Since the resultant is directed vertically downwards, then

$\mathbf{R} = -R\mathbf{k}$

$\because\ \mathbf{R} = \mathbf{T}_{AB} + \mathbf{F} + \mathbf{P}$

$\therefore$ **i**-component gives: $0 = 229.8 + 0.348P - 0.58F$ …… (a)

$\therefore$ **j**-component gives: $0 = -398 + 0.465P$ …… (b)

$\therefore$ **k**-component gives: $-R = -385.67 - 0.814P - 0.814F$ … (c)

From (b) $\Rightarrow P = \boxed{855.9\ \text{N}}$

Substituting into (a) $\Rightarrow$

$$F = \frac{1}{0.58}(0.348 \times 855.9 + 229.8) = \boxed{909.76}\ \text{N}$$

c) *The magnitude of the resultant forces*:

Substituting for *F* and *P* into (c), then

$$R = 0.814\,(855.9\text{ N} + 909.76) + 385.67 = 1822.9 \approx \boxed{1823}\text{ N}$$

d) *The angle between cables AB and AC:*

$cos\ \theta = \mathbf{u}_{AB} \bullet \mathbf{u}_{AC} = (0.383\mathbf{i} - 0.663\mathbf{j} - 0.643\mathbf{k}) \bullet (0.348\mathbf{i} + 0.465\mathbf{j} - 0.814\mathbf{k})$

$$\therefore\ \theta = cos^{-1}(0.348) = \boxed{69.6^{o}}$$

e) *The moment of the tension in cable AB about point C*:

$\mathbf{r}_{CA} = -3\mathbf{i} - 4\mathbf{j} + 7\mathbf{k}$

$$\therefore\ \mathbf{M}_C = \mathbf{r}_{CA} \times \mathbf{T}_{AB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & -4 & 7 \\ 229.9 & -398 & -385.67 \end{vmatrix}$$

$$= \boxed{4328.7\mathbf{i} + 451.6\mathbf{j} + 2113.2\mathbf{k}}$$

Question (6): *(1 · Points)*

When winds blows up (عندما تهب الرياح), the force system in Question (5) changes and may be assumed to be equivalent to a system at point *O* that consists of a resultant force **R** =(25**i**+500**j**+150**k**)N and a resultant moment $\mathbf{M}_O^R$=(300**i**-375**k**)N.m. Determine whether this system can be reduced to a single resultant force. If so, find the point of action of the single resultant. If not, reduce the system into a wrench, then find the pitch and define the axis of wrench.

Solution

Given: **R** =(25**i**+500**j**+150**k**)N and

$\mathbf{M}_O^R$=(300**i**-375**k**)N.m

Since **R. M**^{R} **=** (25**i**+500**j**+150**k**)**.**(300**i**-375**k**)

= (25)(300)+(150)(-375) = -48750 $\neq$ 0

$\therefore$ The system cannot be reduced into a single force. It can be reduced into a *wrench*.

Reduction to a Wrench

The magnitude of **R** is: R =522.6 N

$$\mathbf{u}_R = \frac{\mathbf{R}}{R} = 0.048\mathbf{i} + 0.957\mathbf{j} + 0.287\mathbf{k}.$$

We start by resolving $\mathbf{M}^R$ into two components, one parallel to **R** and the other perpendicular to **R**

$$\therefore\ M_{||} = \mathbf{M}^R.\mathbf{u}_R = (300\mathbf{i}-375\mathbf{k}).\ (0.048\mathbf{i} + 0.957\mathbf{j} + 0.287\mathbf{k})$$

$$= -93.2 \text{ N.m}$$

$$\therefore \text{ The pitch of wrench} = \frac{|M_{||}|}{R} = \frac{93.2}{522.6} = 0.179$$

To specify the axis of wernch:

$$\therefore\ \mathbf{M}_{||} = M_{||}\ \mathbf{u}_R = -93.2\ (0.048\mathbf{i} + 0.957\mathbf{j} + 0.287\mathbf{k})$$

$$= (-4.5\mathbf{i}-89.2\mathbf{j}-26.8\mathbf{k}) \quad \text{N.m}$$

$$\mathbf{M}_\perp = \mathbf{M}^R - \mathbf{M}_{||}$$

$$= (300\mathbf{i}-375\mathbf{k}) - (-4.5\mathbf{i}-89.2\mathbf{j}-26.8\mathbf{k})$$

$$= (304.5\mathbf{i} + 89.2\mathbf{j} - 348.24\mathbf{k}) \quad \text{N.m}$$

Let the wrench pass through the point P : (x, y, 0), then $\mathbf{r}_{OP} = x\mathbf{i} + y\mathbf{j}$ and we require that

$$\mathbf{r}_{OP} \times \mathbf{R} = \mathbf{M}_\perp.$$

$$\therefore \begin{vmatrix} i & j & k \\ x & y & 0 \\ 25 & 500 & 150 \end{vmatrix} = \mathbf{M}_\perp.$$

$\therefore$ $150y\,\mathbf{i} - (150x)\mathbf{j} + (500x - 25y)\mathbf{k} = 304.5\mathbf{i} + 89.2\mathbf{j} - 348.24\mathbf{k}$

Equating the corresponding components yields,

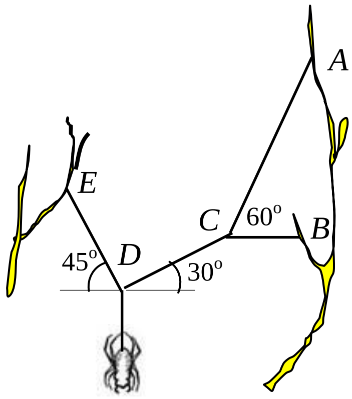
$150y = 304.5; \quad \rightarrow \quad y = 2.03$ m

$-150x = 89.2; \quad \rightarrow \quad x = -0.6$ m

Thus, the axis of wrench is parallel to $\mathbf{u}_R$ and passes through the point (-0.6, 2, 0)

Question (٧): *(10 Points)*

One of the scouts team noticed a spider netting its web on a tree limb in the camping area. Being a freshman Engineering student, the scout estimated that the spider has a mass of 0.7 gram and is suspended from a portion of its web as shown. Determine the force which each of the three web "strings" exerts on the twigs at *A, B,* and *E*. String *CB* is horizontal.



في منطقة المعسكر، لاحظ أحد أعضاء فريق الجوالة عنكبوتاً ينسج عشه علي فرع شجرة. ولأنه طالب في إعدادي هندسة، فقد قدر كتلة العنكبوت المعلق في جزء من شبكته بـ ٠,٧ جرام . فإذا كانت الخيوط المعلق بها العنكبوت تصنع الزوايا المبينة بالشكل، عين القوى التي يؤثر بها كل من الخيوط الثلاثة علي البراعم عند *A*، *B* و *E* مع العلم ان الخيط *CB* افقي.

Solution

Equilibrium of point D:

$\sum F_x = 0;\quad T_1 \cos 30 = T_E \cos 45 \rightarrow \quad T_1 = 0.817 T_E$

$\sum F_y = 0;\quad T_1 \sin 30 + T_E \sin 45 = W;$ substituting for T_1, then

$\therefore\ T_E\,(0.817 \sin 30 + \sin 45) = W$

$$\therefore \quad T_E = \frac{W}{0.817\, sin\, 30 + sin\, 45}$$

$$= \frac{0.7 * 9.8 * 10^{-3}}{1.115}$$

$$= \boxed{6.15(10^{-3})\ \mathrm{N}}$$

$$\therefore \quad T_1 = 0.817 T_E = 5.023\ (10^{-3})\ \mathrm{N}$$

Equilibrium of point C:

$$\sum F_y = 0; \quad T_A\, sin\ 60 = T_1\, sin\ 30 \ \rightarrow$$

$$\boxed{T_A = 2.9*10^{-3}\ \mathrm{N}}$$

$$\sum F_x = 0; \quad T_B + T_A\, cos\ 60 = T_1\, cos\ 30$$

$$\therefore \quad T_B = 5.023(10^{-3})\, cos\ 30 - 2.9*10^{-3}\, cos\ 60$$

$$= \boxed{2.9*10^{-3}\ \mathrm{N}}$$