Mansoura University
Faculty of Engineering
Preparatory Year

Engineering Mechanics
First term (2011-2012)
Time: Two hours

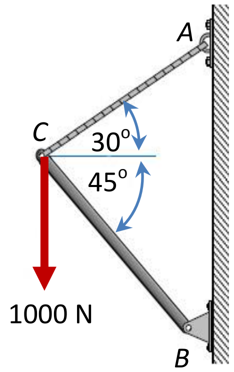
Final Exam

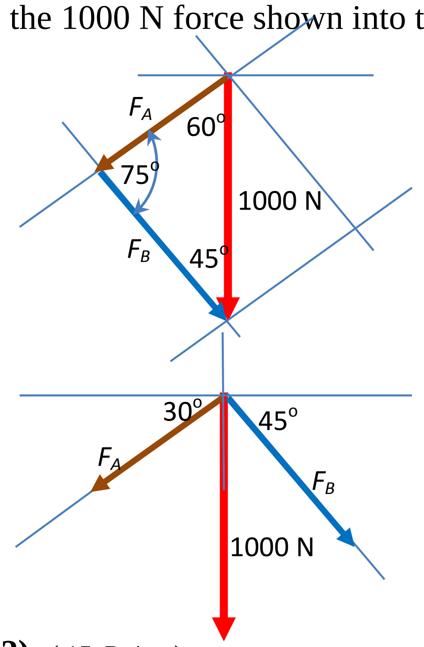
Instructors: Prof. Dr. ***Bishri Abdel-Mo'emen****; Assoc. Prof. Dr.* ***Ayman Ashour****; Dr.* ***Waleed Albeshbeshy***

Answer The Following Questions (Total Mark 90 points)

Question 1: (10 *Points*)

Resolve the 1000 N force shown into two components along *CA* and *CB*.





$$\frac{F_A}{\sin 45} = \frac{F_B}{\sin 60} = \frac{1000}{\sin 75} = 1035.3\ N$$

$$F_A = 1035.3 \sin 45 = 732\ N$$

$$F_B = 1035.3 \sin 60 = 896.6 = 897\ N$$

OR (حل آخر)

$$\sum F_x = 0; \quad F_A \cos 30 = F_B \cos 45$$

$$\therefore F_A = 0.816\, F_B \quad \dots\dots \quad (a)$$

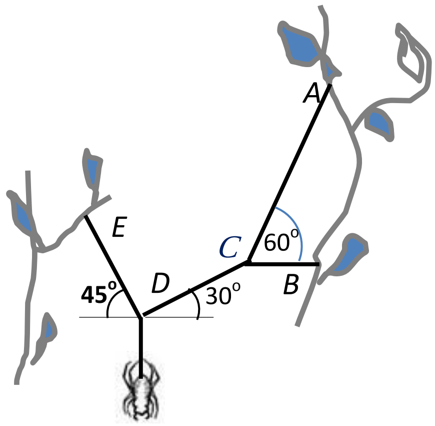
$$\sum F_y = 0; \quad F_A \sin 30 + F_B \sin 45 = 1000$$

Using (a) $\quad \therefore \; 1.115\, F_B = 1000$

$F_B = 896.8 = 987$N and $F_A = 731.8 = 732$ N

Question (2): (15 *Points*)

A spider netting its web on a tree limb. If the spider has a mass of 0.7 gram and is suspended from a portion of its web as shown. Determine the force which each of the three web "strings" exerts on the twigs at *A*, *B*, and *E*. String *CB* is horizontal.



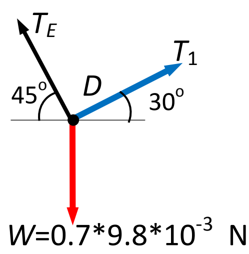
Equilibrium of point D:

$\sum F_x = 0;$ $\quad T_1 \cos 30 = T_E \cos 45 \rightarrow \quad T_1 = 0.817 T_E$

$\sum F_y = 0;$ $\quad T_1 \sin 30 + T_E \sin 45 = W;$ $\quad$ substituting for T_1, then

$\therefore \; T_E\,(0.817 \sin 30 + \sin 45) = W$

$$\therefore \; T_E = \frac{W}{0.817 \sin 30 + \sin 45} = \frac{0.7 * 9.8 * 10^{-3}}{1.115} = \boxed{6.15(10^{-3})\ \text{N}}$$



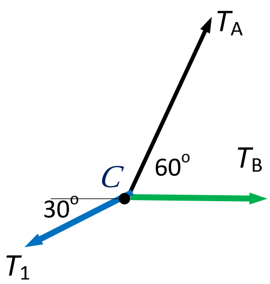
$\therefore \; T_1 = 0.817 T_E = 5.023\ (10^{-3})$ N

Equilibrium of point C:

$\sum F_y = 0;$ $\quad T_A \sin 60 = T_1 \sin 30 \rightarrow \quad \boxed{T_A = 2.9*10^{-3}\ \text{N}}$

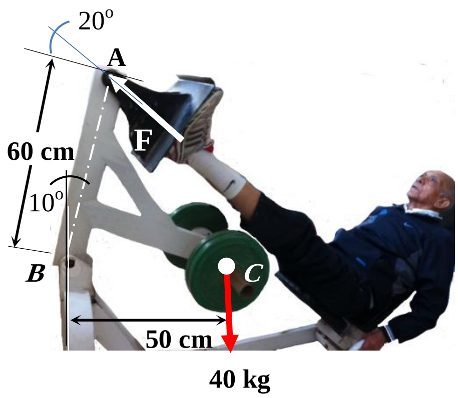
$\sum F_x = 0;$ $\quad T_B + T_A \cos 60 = T_1 \cos 30$

$\therefore \; T_B = 5.023(10^{-3}) \cos 30 - 2.9*10^{-3} \cos 60 = \boxed{2.9*10^{-3}\ \text{N}}$



Question 3: (10 *Points*)

During his Gym routine exercise, Prof. Gonaim exerts a force **F** by his feet on the apparatus in the manner shown in Figure. The combined mass of the load is 40 kg and acts at point *C*. Determine the magnitude of the force *F* act point *A* and the horizontal and vertical components of the reaction at pin *B*.

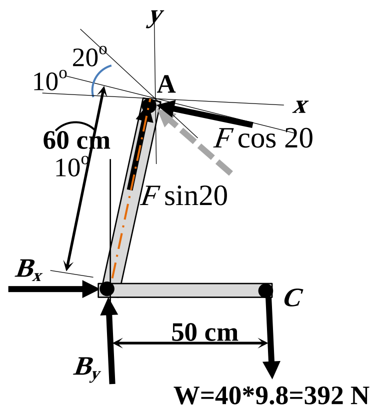


$$\sum M_B = 0; \quad F cos\, 20\,(60) = W(50)$$

$$F = \frac{(392)(50)}{cos\ 20\ (60)} = 347.6\ N \quad Ans.$$

$$\sum F_x = 0; \quad Fcos\ 30 = B_x \quad \rightarrow \quad B_x = 301\ N \quad Ans.$$

$$\sum F_y = 0; \quad Fsin\ 30 + B_y = W \quad \rightarrow \quad B_y = 218\ N \quad Ans.$$



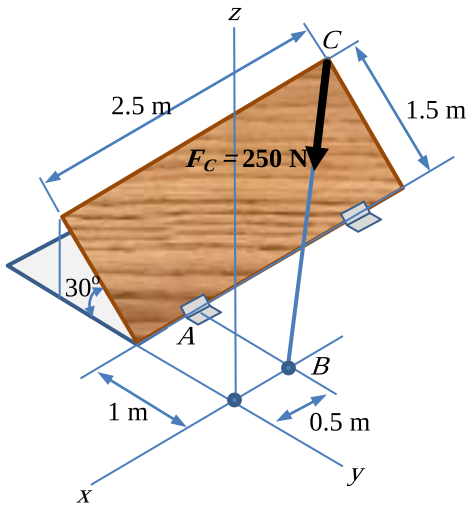
Question 4: (15 *Points*)

A door is kept open by a chain as shown in Figure. Determine:

a. The angle which the force $\mathbf{F}_c$ makes with the direction of *BA*.

b. The moment vector of the force $\mathbf{F}_c$ about the door hinge at *A*.

c. The magnitude of the moment of $\mathbf{F}_c$ about the hinged axis *aa*.



A: (-0.5, -1, 0); *B*: (-0.5, 0, 0); *C*:(-2.5, -2.3, 0.75)

$\mathbf{r}_{AB} = \mathbf{j}; \qquad r_{AB} = \mathbf{1}$

$\mathbf{r}_{CB} = 2\,\mathbf{i} + 2.3\mathbf{j} - 0.75\mathbf{k}; \qquad r_{CB} = \mathbf{3.14}$

$\mathbf{r}_{aa} = \mathbf{i};$

$\mathbf{F}_C = \frac{250}{3.14}(2\,\mathbf{i} + 2.3\mathbf{j} - 0.75\mathbf{k}) = 159.24\,\mathbf{i} + 183.12\mathbf{j} - 59.71\mathbf{k}$

a) $\boldsymbol{\theta} = \boldsymbol{cos}^{-1}\left(\frac{\mathbf{r}_{BA}.\mathbf{r}_{BC}}{r_{BA}r_{CB}}\right) = \boldsymbol{cos}^{-1}\left(\frac{2.3}{3.14}\right) = 43^o$

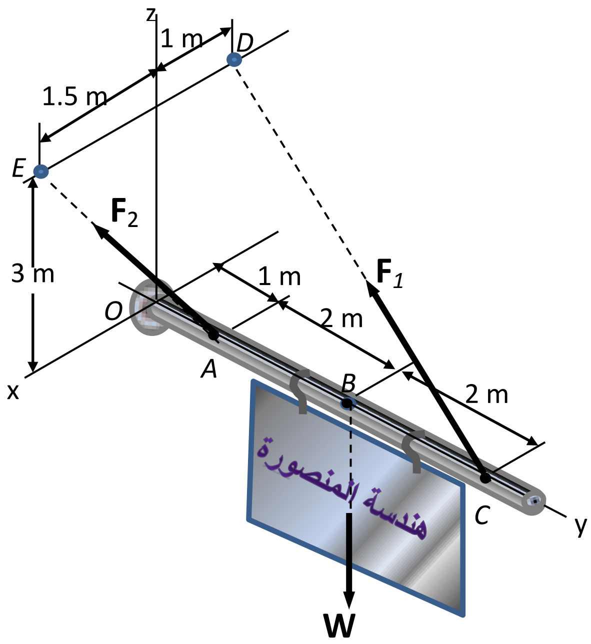
b) $M_A = \mathbf{r}_{AB} \times \mathbf{F}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 0 \\ 159.24 & 183.12 & -59.7 \end{vmatrix}$ = -59.7 **i –** 159.24 **k**

c) $M_A = \mathbf{M}_A.\mathbf{U}_A = (-59.7\,\mathbf{i} - 159.24\,\mathbf{k}).\mathbf{i} = -59.7\ Nm$

Please turn over

Question 5: (15 *Points*)

The two supporting cables exert the forces shown on the sign, where $\mathbf{F_1}$= (-**i**-4.5**j**+3**k**) kN and $\mathbf{F_2}$ = (0.75**i**-0.5**j**+1.5**k**) kN. The weight of the sign is represented by the force **W**= (-6**k**) kN and passes through point *B*. Replace the two forces and the weight by an equivalent force-couple system at point *O*.



Given: $\mathbf{F_1}$= (-**i**-4.5**j**+3**k**) kN

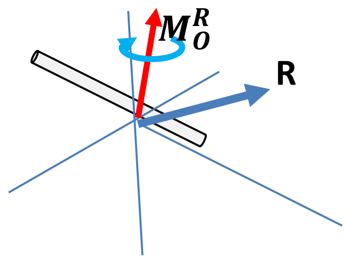
$\mathbf{F_2}$ = (0.75**i**-0.5**j**+1.5**k**) kN

W= (-6**k**) kN

C: (0, 5, 0) , *B*: (0, 3, 0) , *A*: (0, 1, 0)

$\therefore\ \mathbf{r}_{OC} = 5\,\mathbf{j}\,,\ \mathbf{r}_{OB} = 3\mathbf{j},\ \ \mathbf{r}_{OA} = \mathbf{j}$

The Equivalent system at *O*:



$$\mathbf{M}_O^R = \mathbf{r}_{OC} \times \mathbf{F}_1 + \mathbf{r}_{OB} \times \mathbf{W} + \mathbf{r}_{OA} \times \mathbf{F}_2$$

$$= \begin{vmatrix} i & j & k \\ 0 & 5 & 0 \\ -1 & -4.5 & 3 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 0 & 3 & 0 \\ 0 & 0 & -6 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 0 & 1 & 0 \\ 0.75 & -0.5 & 1.5 \end{vmatrix}$$

$$= (15i + 5k) + (-18i) + (1.5i - 0.75k)$$

$$= (-1.5\mathbf{i} + 4.25\mathbf{k})\ \text{kN.m} \qquad \textit{Ans.}$$

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{W} + \mathbf{F}_2$$

= (-**i**-4.5**j**+3**k**) +(-6**k**) +(0.75**i**-0.5**j**+1.5**k**)

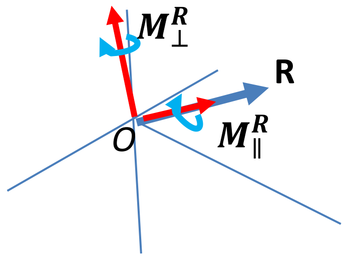
= **(- 0.25i** - 5**j** – 1.5**k**) kN. *Ans.*

Question 6: (15 *Points*)

Assuming that the equivalent force-couple system in Question (7) is given by **R**= (- 250**i** - 5000**j** -1500**k**) N and **M**^{R}= (-1500**i** + 4250**k**) Nm. Reduce this system to a wrench and find its pitch. Specify the point of intersection of the axis of the wrench with *x*-*y* plane.

Given: **R**= (- 250**i** - 5000**j** -1500**k**) N

M^R= (-1500**i** + 4250**k**).



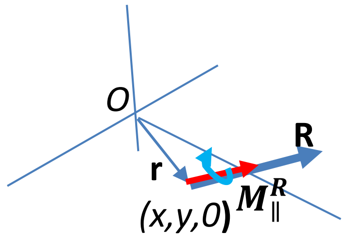
$$\therefore\ R = \sqrt{(250)^2 + (5000)^2 + (1500)^2} = 5226\ N$$

$$\mathbf{u}_R = \frac{\mathbf{R}}{R} = \frac{-250\mathbf{i} - 5000\mathbf{j} - 1500\mathbf{k}}{5226}$$

$$= (-0.048\mathbf{i} - 0.957\mathbf{j} - 0.287\mathbf{k})$$

$$M_\parallel^R = \mathbf{M}^R \cdot \mathbf{u}_R = (-1500)(-0.048) + (4250)(-0.287)$$

$= -1147.8$ N.m (the negative indicates a clockwise rotation when viewed from the tip of **R**)



$$\therefore \mathbf{M}_\parallel^R = M_\parallel^R(\mathbf{u}_R) = (-1147.8)(-0.048\mathbf{i} - 0.957\mathbf{j} - 0.287\mathbf{k})$$

$$= 55\mathbf{i} + 1098.4\mathbf{j} + 329.4\mathbf{k}$$

$$\because \mathbf{M}^R = \mathbf{M}_\perp^R + \mathbf{M}_\parallel^R$$

$$\therefore \mathbf{M}_\perp^R = \mathbf{M}^R - \mathbf{M}_\parallel^R = (-1500\mathbf{i} + 4250\mathbf{k}) - (55\mathbf{i} + 1098.4\mathbf{j} + 329.4\mathbf{k})$$

$$\therefore \mathbf{M}_\perp^R = (-1555\mathbf{i} - 1098.4\mathbf{j} + 3920.6\mathbf{k})$$

Moving the line of action of **R** to a point (x, y, 0), such that

$$\mathbf{r} \times \mathbf{R} = \mathbf{M}_\perp^R$$

$$\therefore \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & y & 0 \\ 250 & 5000 & 1500 \end{vmatrix} = -1555\mathbf{i} - 1098.4\mathbf{j} + 3920.6\mathbf{k}$$

$$\therefore\ 1500y\mathbf{i} - 1500x\mathbf{j} + (5000x - 250y)\mathbf{k} = -1555\mathbf{i} - 1098.4\mathbf{j} + 3920.6\mathbf{k}$$

i-component: $1500\,y = -1555 \ \rightarrow y = -1.04\ m$

j-component: $-1500\,x = -1098.4 \ \rightarrow x = 0.73\ m$

Check: These values satisfy the **K** components (please consider the round off errors)

The wrench consists of : the force **R**= (- 250**i** - 5000**j** -1500**k**) N, and

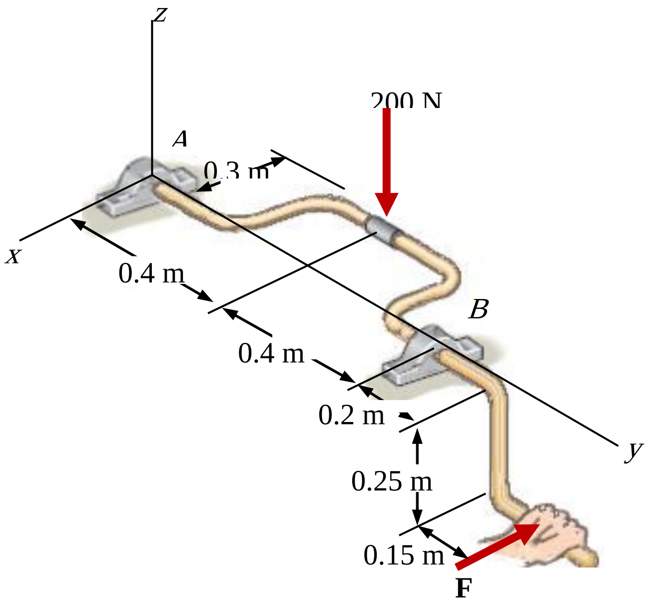
the Moment: $\mathbf{M}_\parallel^R = 55\mathbf{i} + 1098.4\mathbf{j} + 329.4\mathbf{k}$

and act at the point (0.73, -1.04, 0)

The **Pitch** of wrench, P = $\frac{M_\parallel^R}{R} = \frac{-1147.8}{5226}$ = - 0.22 m (the pitch = 22 cm, the sign indicates measuring direction)

Question 7: (10 *Points*)

A vertical force of 200 N acts on the crankshaft. Determine the horizontal force **F** that must be applied to the handle to maintain equilibrium. Determine also components of the reaction forces at the smooth bearing *A* and the thrust bearing *B*. The bearings are properly aligned.



Summing the moment about y-axis:

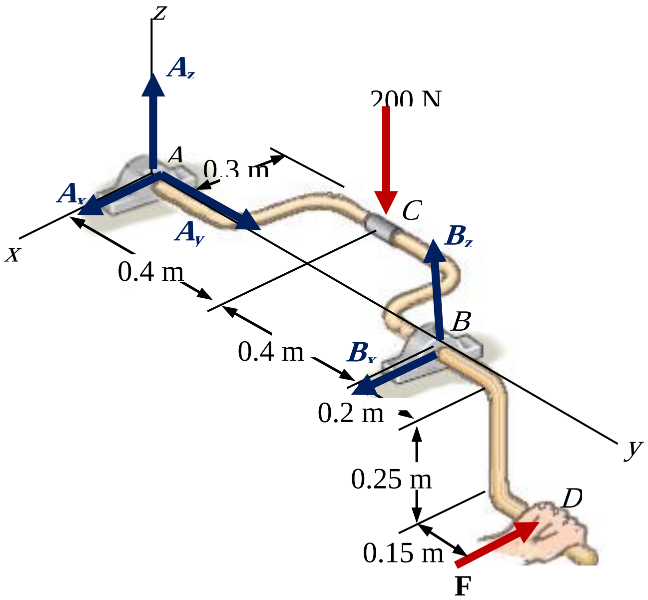
200 (0.3) =*F* (0.25) *F*=240 N

Summing the moment about x-axis:

B_z (0.8)-200(0.4) =0 B_z = 100 N

Summing the moment about z-axis:

B_x (0.8) =*F* (1.0) B_x = 300 N



$$\sum F_x = 0; \quad A_x + B_x - F = 0 \;\rightarrow\; A_x = -60\ N$$

$$\sum F_y = 0; \quad A_y = 0$$

$$\sum F_z = 0; \quad A_z + B_z - 200 = 0 \;\rightarrow\; A_z = 100\ N$$

OR (حل آخر)

C: (-0.3, 0.4, 0) ; *B:* (0, 0.8, 0); *D:*(0, 1.15, -0.25)

$$\sum M_O = 0; \quad \mathbf{r}_C \times (-200\,\mathbf{k}) + \mathbf{r}_B \times \mathbf{B} + \mathbf{r}_D \times (-F\,\mathbf{i}) = 0$$

$$\therefore \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.3 & 0.4 & 0 \\ 0 & 0 & -200 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.8 & 0 \\ B_x & 0 & B_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1.15 & -0.25 \\ -F & 0 & 0 \end{vmatrix} = 0$$

$$\therefore\ (-80\mathbf{i} - 60\mathbf{j}\,) + (0.8\,B_z\mathbf{i} - 0.8\,B_x\mathbf{k}\,) + (0.25\,F\mathbf{j} + 1.15\,F\mathbf{k}\,) = 0$$

$$\therefore\ \mathbf{i} - component \;\rightarrow -80 + 0.8\,B_z = 0 \;\rightarrow\; B_z = 100\ \text{N}$$

$$\therefore\ \mathbf{j} - component \;\rightarrow\; -60 + 0.25\,F = 0 \;\rightarrow\; F = 240\ \text{N}$$

$$\therefore\ \mathbf{k} - component \;\rightarrow\; -0.8B_x + 1.15\,F = 0 \;\rightarrow\; B_x = 345\ \text{N}$$

$$\sum F_x = 0; \quad A_x + B_x - F = 0 \;\rightarrow\; A_x = -60\ N$$

$$\sum F_y = 0; \quad A_y = 0$$

$$\sum F_z = 0; \quad A_z + B_z - 200 = 0 \;\rightarrow\; A_z = 100\ N$$

مع تحياتي وتمنياتي الطيبة......

أ.د. بشري عبد المؤمن (٢٠ يناير ٢٠١٢)